

Title

nlcom — Nonlinear combinations of estimators

Syntax

Nonlinear combination of estimators—one expression

```
nlcom [name:]exp [, options]
```

Nonlinear combinations of estimators—more than one expression

```
nlcom ([name:]exp) ([(name:]exp [, options])
```

<i>options</i>	Description
<u>level</u> (#)	set confidence level; default is level(95)
<u>iterate</u> (#)	maximum number of iterations
<u>post</u>	post estimation results
<u>display_options</u>	control column formats and line width
<u>noheader</u>	suppress output header

`noheader` does not appear in the dialog box.

The second syntax means that if more than one expression is specified, each must be surrounded by parentheses. The optional *name* is any valid Stata name and labels the transformations.

exp is a possibly nonlinear expression containing

```
_b [coef]  
_b [eqno:coef]  
[eqno]coef  
[eqno]_b [coef]
```

eqno is

```
##  
name
```

coef identifies a coefficient in the model. *coef* is typically a variable name, a level indicator, an interaction indicator, or an interaction involving continuous variables. Level indicators identify one level of a factor variable and interaction indicators identify one combination of levels of an interaction; see [U] **11.4.3 Factor variables**. *coef* may contain time-series operators; see [U] **11.4.4 Time-series varlists**.

Distinguish between [], which are to be typed, and [], which indicate optional arguments.

Menu

Statistics > Postestimation > Nonlinear combinations of estimates

Description

`nlcom` computes point estimates, standard errors, test statistics, significance levels, and confidence intervals for (possibly) nonlinear combinations of parameter estimates after any Stata estimation command. Results are displayed in the usual table format used for displaying estimation results. Calculations are based on the “delta method”, an approximation appropriate in large samples.

`nlcom` can be used with `svy` estimation results; see [SVY] [svy postestimation](#).

Options

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [U] [20.7 Specifying the width of confidence intervals](#).

`iterate(#)` specifies the maximum number of iterations used to find the optimal step size in calculating numerical derivatives of the transformation(s) with respect to the original parameters. By default, the maximum number of iterations is 100, but convergence is usually achieved after only a few iterations. You should rarely have to use this option.

`post` causes `nlcom` to behave like a Stata estimation (`eclass`) command. When `post` is specified, `nlcom` will post the vector of transformed estimators and its estimated variance–covariance matrix to `e()`. This option, in essence, makes the transformation permanent. Thus you could, after `posting`, treat the transformed estimation results in the same way as you would treat results from other Stata estimation commands. For example, after `posting`, you could redisplay the results by typing `nlcom` without any arguments, or use `test` to perform simultaneous tests of hypotheses on linear combinations of the transformed estimators; see [R] [test](#).

Specifying `post` clears out the previous estimation results, which can be recovered only by refitting the original model or by storing the estimation results before running `nlcom` and then restoring them; see [R] [estimates store](#).

display_options: `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [estimation options](#).

The following option is available with `nlcom` but is not shown in the dialog box:

`noheader` suppresses the output header.

Remarks

Remarks are presented under the following headings:

Introduction

Basics

Using the post option

Reparameterizing ML estimators for univariate data

nlcom versus eform

Introduction

`nlcom` and `predictnl` both use the delta method. They take nonlinear transformations of the estimated parameter vector from some fitted model and apply the delta method to calculate the variance, standard error, Wald test statistic, etc., of the transformations. `nlcom` is designed for functions of the parameters, and `predictnl` is designed for functions of the parameters and of the data, that is, for predictions.

`nlcom` generalizes `lincom` (see [R] [lincom](#)) in two ways. First, `nlcom` allows the transformations to be nonlinear. Second, `nlcom` can be used to simultaneously estimate many transformations (whether linear or nonlinear) and to obtain the estimated variance–covariance matrix of these transformations.

Basics

In [R] [lincom](#), the following regression was performed:

```
. use http://www.stata-press.com/data/r12/regress
. regress y x1 x2 x3
```

Source	SS	df	MS			
Model	3259.3561	3	1086.45203	Number of obs =	148	
Residual	1627.56282	144	11.3025196	F(3, 144) =	96.12	
Total	4886.91892	147	33.2443464	Prob > F =	0.0000	
				R-squared =	0.6670	
				Adj R-squared =	0.6600	
				Root MSE =	3.3619	

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
x1	1.457113	1.07461	1.36	0.177	-.666934	3.581161
x2	2.221682	.8610358	2.58	0.011	.5197797	3.923583
x3	-.006139	.0005543	-11.08	0.000	-.0072345	-.0050435
_cons	36.10135	4.382693	8.24	0.000	27.43863	44.76407

Then `lincom` was used to estimate the difference between the coefficients of `x1` and `x2`:

```
. lincom _b[x2] - _b[x1]
( 1) - x1 + x2 = 0
```

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
(1)	.7645682	.9950282	0.77	0.444	-1.20218	2.731316

It was noted, however, that nonlinear expressions are not allowed with `lincom`:

```
. lincom _b[x2]/_b[x1]
not possible with test
r(131);
```

Nonlinear transformations are instead estimated using `nlcom`:

```
. nlcom _b[x2]/_b[x1]
      _nl_1:  _b[x2]/_b[x1]
```

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
_nl_1	1.524714	.9812848	1.55	0.122	-.4148688	3.464297

□ Technical note

The notation `_b[name]` is the standard way in Stata to refer to regression coefficients; see [U] 13.5 Accessing coefficients and standard errors. Some commands, such as `lincom` and `test`, allow you to drop the `_b[]` and just refer to the coefficients by *name*. `nlcom`, however, requires the full specification `_b[name]`. □

Returning to our linear regression example, `nlcom` also allows simultaneous estimation of more than one combination:

```
. nlcom (_b[x2]/_b[x1]) (_b[x3]/_b[x1]) (_b[x3]/_b[x2])
      _nl_1:  _b[x2]/_b[x1]
      _nl_2:  _b[x3]/_b[x1]
      _nl_3:  _b[x3]/_b[x2]
```

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
_nl_1	1.524714	.9812848	1.55	0.122	-.4148688	3.464297
_nl_2	-.0042131	.0033483	-1.26	0.210	-.0108313	.002405
_nl_3	-.0027632	.0010695	-2.58	0.011	-.0048772	-.0006493

We can also label the transformations to produce more informative names in the estimation table:

```
. nlcom (ratio21:_b[x2]/_b[x1]) (ratio31:_b[x3]/_b[x1]) (ratio32:_b[x3]/_b[x2])
      ratio21:  _b[x2]/_b[x1]
      ratio31:  _b[x3]/_b[x1]
      ratio32:  _b[x3]/_b[x2]
```

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
ratio21	1.524714	.9812848	1.55	0.122	-.4148688	3.464297
ratio31	-.0042131	.0033483	-1.26	0.210	-.0108313	.002405
ratio32	-.0027632	.0010695	-2.58	0.011	-.0048772	-.0006493

`nlcom` saves the vector of estimated combinations and its estimated variance–covariance matrix in `r()`.

```
. matrix list r(b)
r(b) [1,3]
      ratio21  ratio31  ratio32
c1  1.5247143  -.00421315  -.00276324
. matrix list r(V)
symmetric r(V) [3,3]
      ratio21  ratio31  ratio32
ratio21  .96291982
ratio31  -.00287781  .00001121
ratio32  -.00014234  2.137e-06  1.144e-06
```

Using the post option

When used with the `post` option, `nlcom` saves the estimation vector and variance–covariance matrix in `e()`, making the transformation permanent:

```
. quietly nlcom (ratio21:_b[x2]/_b[x1]) (ratio31:_b[x3]/_b[x1])
> (ratio32:_b[x3]/_b[x2]), post
. matrix list e(b)
e(b)[1,3]
      ratio21      ratio31      ratio32
y1  1.5247143  -.00421315  -.00276324
. matrix list e(V)
symmetric e(V)[3,3]
      ratio21      ratio31      ratio32
ratio21  .96291982
ratio31  -.00287781  .00001121
ratio32  -.00014234  2.137e-06  1.144e-06
```

After posting, we can proceed as if we had just run a Stata estimation (`eclass`) command. For instance, we can replay the results,

```
. nlcom
```

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
ratio21	1.524714	.9812848	1.55	0.122	-.4148688	3.464297
ratio31	-.0042131	.0033483	-1.26	0.210	-.0108313	.002405
ratio32	-.0027632	.0010695	-2.58	0.011	-.0048772	-.0006493

or perform other postestimation tasks in the transformed metric, this time making reference to the new “coefficients”:

```
. display _b[ratio31]
-.00421315
. estat vce, correlation
Correlation matrix of coefficients of nlcom model
      e(V) | ratio21  ratio31  ratio32
-----|-----
ratio21 | 1.0000
ratio31 | -0.8759  1.0000
ratio32 | -0.1356  0.5969  1.0000
. test _b[ratio21] = 1
( 1)  ratio21 = 1
      F( 1, 144) = 0.29
      Prob > F = 0.5937
```

We see that testing `_b[ratio21]=1` in the transformed metric is equivalent to testing using `testnl _b[x2]/_b[x1]=1` in the original metric:

```
. quietly reg y x1 x2 x3
. testnl _b[x2]/_b[x1] = 1
(1)  _b[x2]/_b[x1] = 1
      F(1, 144) = 0.29
      Prob > F = 0.5937
```

We needed to refit the regression model to recover the original parameter estimates.

□ **Technical note**

In a previous technical note, we mentioned that commands such as `lincom` and `test` permit reference to *name* instead of `_b[name]`. This is not the case when `lincom` and `test` are used after `nlcom`, `post`. In the above, we used

```
. test _b[ratio21] = 1
```

rather than

```
. test ratio21 = 1
```

which would have returned an error. Consider this a limitation of Stata. For the shorthand notation to work, you need a variable named *name* in the data. In `nlcom`, however, *name* is just a coefficient label that does not necessarily correspond to any variable in the data. □

Reparameterizing ML estimators for univariate data

When run using only a response and no covariates, Stata's maximum likelihood (ML) estimation commands will produce ML estimates of the parameters of some assumed univariate distribution for the response. The parameterization, however, is usually not one we are used to dealing with in a nonregression setting. In such cases, `nlcom` can be used to transform the estimation results from a regression model to those from a maximum likelihood estimation of the parameters of a univariate probability distribution in a more familiar metric.

▷ Example 1

Consider the following univariate data on $Y = \#$ of traffic accidents at a certain intersection in a given year:

```
. use http://www.stata-press.com/data/r12/trafint
. summarize accidents
```

Variable	Obs	Mean	Std. Dev.	Min	Max
accidents	12	13.83333	14.47778	0	41

A quick glance of the output from `summarize` leads us to quickly reject the assumption that Y is distributed as Poisson because the estimated variance of Y is much greater than the estimated mean of Y .

Instead, we choose to model the data as univariate negative binomial, of which a common parameterization is

$$\Pr(Y = y) = \frac{\Gamma(r + y)}{\Gamma(r)\Gamma(y + 1)} p^r (1 - p)^y \quad 0 \leq p \leq 1, \quad r > 0, \quad y = 0, 1, \dots$$

with

$$E(Y) = \frac{r(1 - p)}{p} \quad \text{Var}(Y) = \frac{r(1 - p)}{p^2}$$

There exist no closed-form solutions for the maximum likelihood estimates of p and r , yet they may be estimated by the iterative method of Newton–Raphson. One way to get these estimates would be to write our own Newton–Raphson program for the negative binomial. Another way would be to write our own ML evaluator; see [R] `ml`.

The easiest solution, however, would be to use Stata's existing negative binomial ML regression command, `nbreg`. The only problem with this solution is that `nbreg` estimates a different parameterization of the negative binomial, but we can worry about that later.

```
. nbreg accidents
Fitting Poisson model:
Iteration 0:   log likelihood = -105.05361
Iteration 1:   log likelihood = -105.05361
Fitting constant-only model:
Iteration 0:   log likelihood = -43.948619
Iteration 1:   log likelihood = -43.891483
Iteration 2:   log likelihood = -43.89144
Iteration 3:   log likelihood = -43.89144
Fitting full model:
Iteration 0:   log likelihood = -43.89144
Iteration 1:   log likelihood = -43.89144
Negative binomial regression
Dispersion      = mean
Log likelihood  = -43.89144
Number of obs   =          12
LR chi2(0)     =           0.00
Prob > chi2    =           .
Pseudo R2      =          0.0000
```

accidents	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
_cons	2.627081	.3192233	8.23	0.000	2.001415	3.252747
/lnalpha	.1402425	.4187147			-.6804233	.9609083
alpha	1.150553	.4817534			.5064026	2.61407

Likelihood-ratio test of alpha=0: `chibar2(01) = 122.32 Prob>=chibar2 = 0.000`

```
. nbreg, coeflegend
Negative binomial regression
Dispersion      = mean
Log likelihood  = -43.89144
Number of obs   =          12
LR chi2(0)     =           0.00
Prob > chi2    =           .
Pseudo R2      =          0.0000
```

accidents	Coef.	Legend
_cons	2.627081	_b[accidents:_cons]
/lnalpha	.1402425	_b[lnalpha:_cons]
alpha	1.150553	

Likelihood-ratio test of alpha=0: `chibar2(01) = 122.32 Prob>=chibar2 = 0.000`

From this output, we see that, when used with univariate data, `nbreg` estimates a regression intercept, β_0 , and the logarithm of some parameter α . This parameterization is useful in regression models: β_0 is the intercept meant to be augmented with other terms of the linear predictor, and α is an overdispersion parameter used for comparison with the Poisson regression model.

However, we need to transform $(\beta_0, \ln\alpha)$ to (p, r) . Examining *Methods and formulas* of [R] `nbreg` reveals the transformation as

$$p = \{1 + \alpha \exp(\beta_0)\}^{-1} \quad r = \alpha^{-1}$$

which we apply using `nlcom`:

```
. nlcom (p:1/(1 + exp([lnalpha]_b[_cons] + _b[_cons])))
> (r:exp(-[lnalpha]_b[_cons]))
      p: 1/(1 + exp([lnalpha]_b[_cons] + _b[_cons]))
      r: exp(-[lnalpha]_b[_cons])
```

accidents	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
p	.0591157	.0292857	2.02	0.044	.0017168	.1165146
r	.8691474	.3639248	2.39	0.017	.1558679	1.582427

Given the invariance of maximum likelihood estimators and the properties of the delta method, the above parameter estimates, standard errors, etc., are precisely those we would have obtained had we instead performed the Newton–Raphson optimization in the (p, r) metric.



□ Technical note

Note how we referred to the estimate of $\ln\alpha$ above as `[lnalpha]_b[_cons]`. This is not entirely evident from the output of `nbreg`, which is why we redisplayed the results using the `coeflegend` option so that we would know how to refer to the coefficients; [U] 13.5 Accessing coefficients and standard errors.



nlcom versus eform

Many Stata estimation commands allow you to display exponentiated regression coefficients, some by default, some optionally. Known as “eform” in Stata terminology, this reparameterization serves many uses: it gives odds ratios for logistic models, hazard ratios in survival models, incidence-rate ratios in Poisson models, and relative-risk ratios in multinomial logit models, to name a few.

For example, consider the following estimation taken directly from the technical note in [R] poisson:

```
. use http://www.stata-press.com/data/r12/airline
. gen lnN = ln(n)
. poisson injuries XYZowned lnN
Iteration 0: log likelihood = -22.333875
Iteration 1: log likelihood = -22.332276
Iteration 2: log likelihood = -22.332276
Poisson regression
Log likelihood = -22.332276
Number of obs = 9
LR chi2(2) = 19.15
Prob > chi2 = 0.0001
Pseudo R2 = 0.3001
```

injuries	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
XYZowned	.6840667	.3895877	1.76	0.079	-.0795111	1.447645
lnN	1.424169	.3725155	3.82	0.000	.6940517	2.154285
_cons	4.863891	.7090501	6.86	0.000	3.474178	6.253603

When we replay results and specify the `irr` (incidence-rate ratios) option,

```
. poisson, irr
Poisson regression                Number of obs   =          9
                                  LR chi2(2)         =         19.15
                                  Prob > chi2        =         0.0001
Log likelihood = -22.332276       Pseudo R2       =         0.3001
```

injuries	IRR	Std. Err.	z	P> z	[95% Conf. Interval]	
XYZowned	1.981921	.7721322	1.76	0.079	.9235678	4.253085
lnN	4.154402	1.547579	3.82	0.000	2.00181	8.621728
_cons	129.5272	91.84126	6.86	0.000	32.2713	519.8828

we obtain the exponentiated regression coefficients and their estimated standard errors.

Contrast this with what we obtain if we exponentiate the coefficients manually by using `nlcom`:

```
. nlcom (E_XYZowned:exp(_b[XYZowned])) (E_lnN:exp(_b[lnN]))
E_XYZowned:  exp(_b[XYZowned])
E_lnN:       exp(_b[lnN])
```

injuries	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
E_XYZowned	1.981921	.7721322	2.57	0.010	.4685701	3.495273
E_lnN	4.154402	1.547579	2.68	0.007	1.121203	7.187602

There are three things to note when comparing `poisson, irr` (and `eform` in general) with `nlcom`:

1. The exponentiated coefficients and standard errors are identical. This is certainly good news.
2. The Wald test statistic (z) and level of significance are different. When using `poisson, irr` and other related `eform` options, the Wald test does not change from what you would have obtained without the `eform` option, and you can see this by comparing both versions of the `poisson` output given previously.

When you use `eform`, Stata knows that what is usually desired is a test of

$$H_0: \exp(\beta) = 1$$

and not the uninformative-by-comparison

$$H_0: \exp(\beta) = 0$$

The test of $H_0: \exp(\beta) = 1$ is asymptotically equivalent to a test of $H_0: \beta = 0$, the Wald test in the original metric, but the latter has better small-sample properties. Thus if you specify `eform`, you get a test of $H_0: \beta = 0$.

`nlcom`, however, is general. It does not attempt to infer the test of greatest interest for a given transformation, and so a test of

$$H_0: \text{transformed coefficient} = 0$$

is always given, regardless of the transformation.

3. You may be surprised to see that, even though the coefficients and standard errors are identical, the confidence intervals (both 95%) are different.

`eform` confidence intervals are standard confidence intervals with the endpoints transformed. For example, the confidence interval for the coefficient on `lnN` is $[0.694, 2.154]$, whereas the confidence interval for the incidence-rate ratio due to `lnN` is $[\exp(0.694), \exp(2.154)] = [2.002, 8.619]$, which, except for some roundoff error, is what we see from the output of `poisson, irr`. For exponentiated coefficients, confidence intervals based on transform-the-endpoints methodology generally have better small-sample properties than their asymptotically equivalent counterparts.

The transform-the-endpoints method, however, gives valid coverage only when the transformation is monotonic. `nlcom` uses a more general and asymptotically equivalent method for calculating confidence intervals, as described in *Methods and formulas*.

Saved results

`nlcom` saves the following in `r()`:

Scalars

`r(N)` number of observations
`r(df_r)` residual degrees of freedom

Matrices

`r(b)` vector of transformed coefficients
`r(V)` estimated variance–covariance matrix of the transformed coefficients

If `post` is specified, `nlcom` also saves the following in `e()`:

Scalars

`e(N)` number of observations
`e(df_r)` residual degrees of freedom
`e(N_strata)` number of strata L , if used after `svy`
`e(N_psu)` number of sampled PSUs n , if used after `svy`
`e(rank)` rank of `e(V)`

Macros

`e(cmd)` `nlcom`
`e(predict)` program used to implement `predict`
`e(properties)` `b V`

Matrices

`e(b)` vector of transformed coefficients
`e(V)` estimated variance–covariance matrix of the transformed coefficients
`e(V_srs)` simple-random-sampling-without-replacement (co)variance $\widehat{V}_{\text{srswoR}}$, if `svy`
`e(V_srswr)` simple-random-sampling-with-replacement (co)variance $\widehat{V}_{\text{srswr}}$, if `svy` and `fpv()`
`e(V_msp)` misspecification (co)variance \widehat{V}_{msp} , if `svy` and available

Functions

`e(sample)` marks estimation sample

Methods and formulas

`nlcom` is implemented as an ado-file.

Given a $1 \times k$ vector of parameter estimates, $\widehat{\theta} = (\widehat{\theta}_1, \dots, \widehat{\theta}_k)$, consider the estimated p -dimensional transformation

$$g(\widehat{\theta}) = [g_1(\widehat{\theta}), g_2(\widehat{\theta}), \dots, g_p(\widehat{\theta})]$$

The estimated variance–covariance of $g(\widehat{\theta})$ is given by

$$\widehat{\text{Var}} \{g(\widehat{\theta})\} = \mathbf{G} \mathbf{V} \mathbf{G}'$$

where \mathbf{G} is the $p \times k$ matrix of derivatives for which

$$\mathbf{G}_{ij} = \left. \frac{\partial g_i(\boldsymbol{\theta})}{\partial \theta_j} \right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} \quad i = 1, \dots, p \quad j = 1, \dots, k$$

and \mathbf{V} is the estimated variance–covariance matrix of $\hat{\boldsymbol{\theta}}$. Standard errors are obtained as the square roots of the variances.

The Wald test statistic for testing

$$H_0: g_i(\boldsymbol{\theta}) = 0$$

versus the two-sided alternative is given by

$$Z_i = \frac{g_i(\hat{\boldsymbol{\theta}})}{\left[\widehat{\text{Var}}_{ii} \left\{ g(\hat{\boldsymbol{\theta}}) \right\} \right]^{1/2}}$$

When the variance–covariance matrix of $\hat{\boldsymbol{\theta}}$ is an asymptotic covariance matrix, Z_i is approximately distributed as Gaussian. For linear regression, Z_i is taken to be approximately distributed as $t_{1,r}$ where r is the residual degrees of freedom from the original fitted model.

A $(1 - \alpha) \times 100\%$ confidence interval for $g_i(\boldsymbol{\theta})$ is given by

$$g_i(\hat{\boldsymbol{\theta}}) \pm z_{\alpha/2} \left[\widehat{\text{Var}}_{ii} \left\{ g(\hat{\boldsymbol{\theta}}) \right\} \right]^{1/2}$$

for those cases where Z_i is Gaussian and

$$g_i(\hat{\boldsymbol{\theta}}) \pm t_{\alpha/2,r} \left[\widehat{\text{Var}}_{ii} \left\{ g(\hat{\boldsymbol{\theta}}) \right\} \right]^{1/2}$$

for those cases where Z_i is t distributed. z_p is the $1 - p$ quantile of the standard normal distribution, and $t_{p,r}$ is the $1 - p$ quantile of the t distribution with r degrees of freedom.

References

- Feiveson, A. H. 1999. FAQ: What is the delta method and how is it used to estimate the standard error of a transformed parameter? <http://www.stata.com/support/faqs/stat/deltam.html>.
- Gould, W. W. 1996. crc43: Wald test of nonlinear hypotheses after model estimation. *Stata Technical Bulletin* 29: 2–4. Reprinted in *Stata Technical Bulletin Reprints*, vol. 5, pp. 15–18. College Station, TX: Stata Press.
- Oehlert, G. W. 1992. A note on the delta method. *American Statistician* 46: 27–29.
- Phillips, P. C. B., and J. Y. Park. 1988. On the formulation of Wald tests of nonlinear restrictions. *Econometrica* 56: 1065–1083.

Also see

- [R] **lincom** — Linear combinations of estimators
- [R] **predictnl** — Obtain nonlinear predictions, standard errors, etc., after estimation
- [R] **test** — Test linear hypotheses after estimation
- [R] **testnl** — Test nonlinear hypotheses after estimation
- [U] **20 Estimation and postestimation commands**